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LMI-based feedback regulator design via TP transformation for fluid volume control in blood purification therapies

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***Abstract*— With blood purification the life of a person suffering by kidney malfunction can be saved or the quality of her/his life can be increased. This is the purpose of hemodialysis machines, where the blood of the patient is filtered (cleared) in an extracorporeal tube system. Here peristaltic pumps are responsible for the fluid transport, where the strict control is a serious need, in order to maintain patient fluid balance and meet exact dosage. The current paper focuses on designing a robust controller with TP transformation. This controller is presented from the application point of view. The experiences with the designed controller are tested on a real system discussing their applicability.**

Keywords-TP transformation, LMI-based control design, blood purification

I. Introduction

According to prediction [1] the number of patients with end stage renal disease (ESRD) is increasing. This means that the need for kidney transplantation and blood purification methods will be growing. In case of ESRD and some other kidney malfunction the organ is not capable to perform its excretion function partially or completely. In case of kidney (excretion) insufficiency the final solution is kidney transplantation to restore the organ's function. However, the necessary donor organ is usually unavailable. Until a suitable organ's availability the blood of the patient is treated with hemodialysis machine. This machine filters (cleans) the blood of the patient in an extracorporeal tube system through a semi-permeable membrane. In such machines the transportation of blood and other fluids, such as ultrafiltrate, are done with peristaltic pumps, [2, 3, 4]. The transferred fluid remains intact as it flows in disinfected, disposable tubing. Furthermore, the peristaltic pumps exert lower shearing forces on the transported liquids, which means reduced chance of hemolysis in case of blood transport [5].

The periodic compression of an elastic pump segment is responsible for the fluid transport in the peristaltic pumps. The compression is created between the rollers on the central rotor and the housing (called manifold). The rollers move in such a way, that the tubing is closed in every moment by at least one roller; this operation prevents the back-flow. On the other hand, during compression the roller pushes the fluid in front of itself creating a pressure wave in the tubing and realizing the fluid transport [6].

The accuracy of these pumps mainly depends on the accuracy of the volume of the tube segment. The most relevant factor in the inaccuracy is the deviation of the segment production. This may cause difference in transferred fluid volume up to ±10% compared to the expected one [7]. Due to the repeated compression the material of the tube segment may fatigue. However, this process is slow (noticeable in tens of hours). Therefore, this component can be neglected in short term measurements, but it has to be considered on long term processes. In these cases the error is more relevant at higher flow rates. Also the pressure in the tubing precedes the tube segment, it influences the transport and has a nonlinear correlation. As these pumps are responsible for the transfer of blood, drugs, medical fluids and for the patient fluid balance, the strict control of the pumps is inevitable [5, 8]. As the treated patients usually have no or minimal urine excretion, the machine is responsible for the removal of unnecessary water. This also accentuates the importance of control.

In previous papers the behavior of the corresponding subsystem (weighting scale, tubing, peristaltic pump, elastic tube segment and final reservoir) was identified [9]. Furthermore, a fuzzy system and an adaptive neuro-fuzzy network were designed and compared [9, 10, 11].

The goal of current work is to create a robust controller that is fast and lightweight and works over a wide flow range (approx. two decades), while it is capable tolerating the changes of the tube segment. The designed controller compared to the previous work results [9, 10, 11].

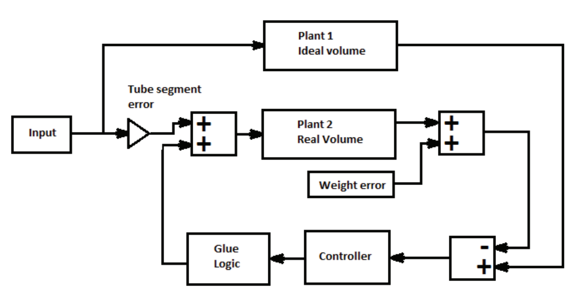

Figure 1. Schematic diagram of the examined model

II. Method

A. Former results

The relevant subsystem was identified [9]:

$$v_{\text{ideal}}(t + T_S) = v_{\text{ideal}}(t) + K u_{0nom}\,, \tag{1}$$

where u_{0nom} is the nominal flow and v is the transferred fluid volume, K parameter defines the transfer and T_s is the sampling period (250 ms).

The system:

$$v_{real}(t + T_S) = v_{real}(t) + K\, u_{0real}\,(1 + u), \tag{2}$$

where $u_{0real} = u_{0nom} + u_{0error}$ and u is limited by the system requirements.

The pressure dependence is neglected at first, as it is more practical to correct it with a feed-forward control, while for the other parts a feedback control is designed.

The time dependence of the K parameter is also neglected as the change during a standard therapy is not relevant.

The structure of the model is based on the original one [9], whit the modification introduced in [11]. Here, the model comprises two branches, both with the same plant. The first branch realizes an ideal behavior, where the ideal transfer volume is calculated. In the second branch the real transfer volume is created by introducing the offset and slope errors. The schematic of the applied model is presented in Fig. 1.

The possible error includes slope error, which simulates the volume error of the tube segment. With constant error the deviation of production can be simulated, while a slow dynamic change can be introduced to mimic the fatigue. A static offset error can simulate accumulated volume error in the system. The dynamic offset error can simulate unexpected environmental effects, such as the partial block of tubing, movement of weighting scale or other disturbances.

The error signal is created by subtracting the real transferred volume from the real transferred volume. Then, the control signal is created and fed back to the system. It is important to note, that the control signal is limited on the one hand due to the hardware issues that prohibit the use of arbitrary control signal. On the other hand –and this is a stricter limit– there are system requirements, based on given standards that strongly limit the magnitude of the control signal. This saturation is embedded in the model.

B. Controller design

The following error system can be concerned to design a controller [12, 13, 14] by creating $\Delta v = v_{real} - v_{ideal}$:

$$\Delta v(t + T_S) = \Delta v(t) + K\, u_{0real}\, u + K\, u_{0error} \tag{3}$$

where the goal of the control is to reach zero state.

The main idea is to design a PI controller, as the proportional component is capable to eliminate the error of the tube segment (slope error), while the integrator is responsible to eliminate the other error components, especially the small ones by accumulation. Discrete controller has to be designed, as the controller is applied in a real machine.

The control input of the PI controller can be determined as follows:

$$u = -\begin{bmatrix} F_1 & F_2 \end{bmatrix} \begin{bmatrix} \Delta v(+z) \\ \sum(\Delta v(+z)) T_S \end{bmatrix} \tag{4}$$

where z is the measurement error and the integrator eliminates the effect K u_{0error} of (3),), F_1 and F_2 are the controller's parts.

Equation (3) expanded with the integral of Δv results:

$$\begin{bmatrix} \Delta v(t + T_S) \\ \sum_0^{t+T_S} \Delta v T_S \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ T_S & 1 \end{bmatrix} \begin{bmatrix} \Delta v(t) \\ \sum_0^{t} \Delta v T_S \end{bmatrix} + \begin{bmatrix} K u_{0real} \\ 0 \end{bmatrix} u + \begin{bmatrix} K u_{0error} \\ 0 \end{bmatrix} \tag{5}$$

The goal is to create complete state feedback to reach zero state control. The present paper targets to prove that control design based on Linear Matrix Inequalities (LMI) techniques is effective in safety-critical systems. First, Tensor Product (TP) model transformation is applied at [15, 16, 17, 18].

The Linear Parameter Varying (LPV) model can be written as:

$$\boldsymbol{x}(t + T_S) = \boldsymbol{A}(p(t))\boldsymbol{x}(t) + \boldsymbol{B}(p(t))u(t), \tag{6}$$

where:

- the state variables are:
$$\boldsymbol{x}(t) = \begin{bmatrix} \Delta v(t) & \sum_0^t \Delta v(i) T_S \end{bmatrix}^T \tag{7}$$
- the control signal is $u(t)$
- the scheduling parameter is represented by $p = u_{0real}$,
- the parameter depending state matrices are

$$\boldsymbol{S}(p) = \begin{bmatrix} \boldsymbol{A}(p(t)) & \boldsymbol{B}(p(t)) \end{bmatrix} = \begin{bmatrix} 1 & 0 & K u_{0real} \\ T_S & 1 & 0 \end{bmatrix} \tag{8}$$

The TP model resulted by the execution of the transformation:

$$\boldsymbol{x}(t + T_S) = \boldsymbol{S}(p) \begin{bmatrix} \boldsymbol{x}(t) \\ u(t) \end{bmatrix} \tag{9}$$

where $\boldsymbol{S}(p)$ is given in a convex tensor product form:

$$\boldsymbol{S}(p) = S \times_1 \boldsymbol{w}(p) = \sum_{j=1}^{2} w_j(p) \boldsymbol{S}_j \tag{10}$$

The vertex systems are:

$$\boldsymbol{S}_1 = \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{B}_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & K u_{0realmin} \\ T_S & 1 & 0 \end{bmatrix} \tag{11}$$

and

$$\boldsymbol{S}_2 = \begin{bmatrix} \boldsymbol{A}_2 & \boldsymbol{B}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & K u_{0realmax} \\ T_S & 1 & 0 \end{bmatrix} \tag{12}$$

The resulted weighting functions are shown on Fig. 2.

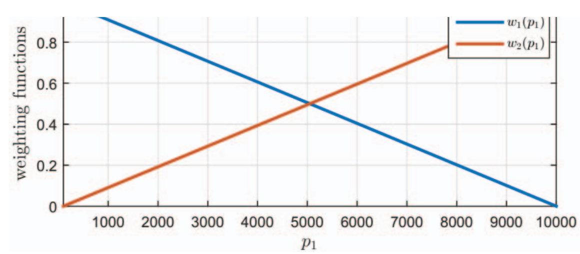

Figure 2. Weighting functions used for TP modeling

For the full state feedback control, the control input is:

$$u(t) = -\boldsymbol{F}\boldsymbol{x}(t). \tag{13}$$

The following lemma was used for the controller design according to [17, 19, 20]:

Lemma 1: The $\mathbf{F}$ controller ensure quadratic stability of system (9) and minimize the v upper bound of the

$$J = \sum_{i=0}^{\infty} \mathbf{x}^T(t_i)\mathbf{W}\mathbf{x}(t_i) < v \tag{14}$$

performance function starting the system from the $\mathbf{x}_0$ state, by solving the following LMI optimization problem:

minimize v

$$\mathbf{x} = \mathbf{x}^T, \mathbf{M} \tag{15}$$

subject to

$$\begin{bmatrix} 1 & \mathbf{x}_0^T \\ \mathbf{x}_0 & \mathbf{X} \end{bmatrix} \geq 0, \text{ and}$$

$$\begin{bmatrix} -\mathbf{X} & (\mathbf{A}_j\mathbf{X} - \mathbf{B}_j\mathbf{M})^T & \mathbf{X} \\ \mathbf{A}_j\mathbf{X} - \mathbf{B}_j\mathbf{M} & -\mathbf{X} & 0 \\ \mathbf{X} & 0 & -v\mathbf{W}^{-1} \end{bmatrix} < 0 \;\forall j \tag{16}$$

As solution, the controller is derived as $\mathbf{F} = \mathbf{M}\mathbf{X}^{-1}$.

In order to numerically solve the above mentioned LMI problem, the YALMIP toolbox [19] and MOSEK solver [20] were used for optimization. The resulted gains in case of $\mathbf{W} = \begin{bmatrix} 1 & 0 \\ 0 & 0.01 \end{bmatrix}$ and $\mathbf{x}_0 = [1 \quad 0]^T$:

$$F = [-2.6465 \quad -0.1447] \tag{15}$$

Anti-windup feedback was applied to the system, as saturation occurs in the system.

III. Results

The quality of the designed controller was examined by the following properties:

- settling time
- overshoot
- accuracy

The settling time was examined by assuming ideal tube segment (no slope error) and measuring how much time does it need to compensate 5 ml error. The idea behind this is that there are some cases, when the system may accumulate volume error. A good example is if the weighting scale is got stuck temporarily for a brief time (e.g. it is pushed against a bed).

This compensation time was measured over the regular flow range of the system (100-1500 ml/h). Also the minimal compensation time was calculated. The average settling time was 1.67 times more as the minimal compensation time (σ = ±0.1). These values are acceptable, as during the calculation of minimal compensation time some issues were intentionally neglected (e.g. system delay, inertia of the pump, etc.). Furthermore, the slower system means, that the machine is gentler to the patient, due to the smoother compensation and the less impulse-like compensations. Altogether, this seems to be an optimal comprise between compensation time and patient needs.

The settling time and the overshoot examination used the same assembly. Here a worst case situation was examined, namely the tube segment had the biggest acceptable error (-10% slope error, less volume transfer), while high, namely 20 ml, offset error was introduced (according to the normal behavior this means a rarely occurring high error during standard operation). The settling time was measured according to the following method: in steady-state the error signal was summarized for 200 seconds.

Overshoot cannot be measured below 1000 ml/h, the maximal overshoot was measured at 1500 ml/h with the value of 3 ml. The overshoot of the system was minimal; hence, it can be neglected. Although this parameter may be seemingly less relevant, it still has a greater importance. The overshoot gives information about the patient burden of the control. The overshoot is a temporary volume error, which is not inherent. This error is introduced by the controller and burdens the patient by administering or removing too much fluid from her/his body. As a result, the 3 ml overshoot is an irrelevant amount compared to the blood volume of a human (approx. 5 l [24]).

The settling time is in correlation with the patient fluid balance. It is inevitable to keep the patient fluid balance (sum of removed and administered volumes), otherwise the patient could dry out or could have too many water in her/his body. To ensure the patient's fluid balance, the long term volume error should tend to zero. To examine this, the integral of the error signal was calculated for a 200 s wide window. The average of sum of error was 2 ml (σ = ± 1 ml). This amount can be neglected compared to the fluid blood volume, also it proves, that there is no error accumulation in the system.

IV. Measurements on the Real System

The designed controller was embedded to the real system with automatic code generation.

The previous measures were examined with the real system as well, by preserving the examination conditions. The average settling time was 1.92 times of the minimal compensation time (σ = ±0.32). This amount is significantly higher compared to those obtained by simulations. The most important reason for this is that during simulation ideal tube segment (with perfect transfer) was assumed. However, in the reality this is unfeasible and the tube segment has a transfer error as well. As the system has to compensate the slope error and the volume error, this phenomena explains the difference. The overshoot was similarly zero at low flow rates, while it appears at high (over 1000 ml/h) and the maximal value of overshoot is 3 ml. The average of sum of error in steady-state was 5 ml (σ = ± 2 ml), which is a small increase, but it can be still neglected.

The error signal of a measurement on the real machine is presented on Fig. 3. The quantitation error presents that the weighting scale is capable to measure the weight as an integer. This situation considered explains the reason why the derivative of the error signal could not be used for controller design; although it would be beneficial.

Further research will focus pn the optimization of the anti-windup system, according to [21], [22]. The pressure dependence is neglected at the moment, but it could be compensated with a feed-forward control. This should be also integrated in the system.

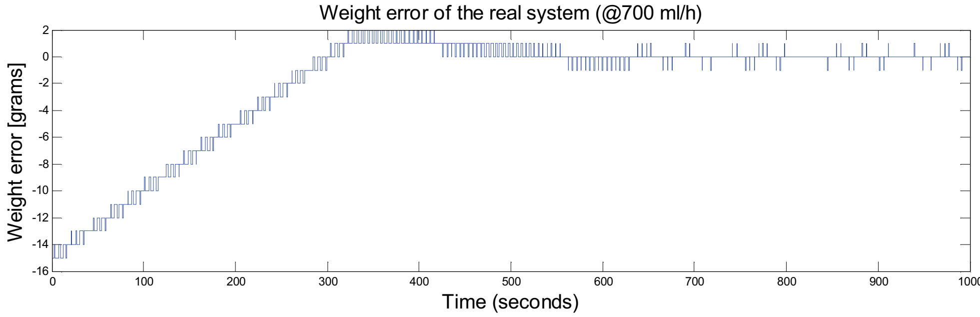

Figure 3. Behavior of the real system for flow rate of 700 ml/h

V. CONCLUSION

The present paper has shown the description of the fluid control system of a hemodialysis machine. The identified model was described in an LPV form. The goal of this paper was to show, that LMI-based control design can be effectively used in such machines. In order to do this TP transformation was applied in the original model and for this a PI controller was designed.

The controller was capable to compensate both the slope error and the volume errors, while it was gentle to the patient. The compensation time could be further decreased, but at the moment the controller is compromised between speed and tenderness. The overflow and the accumulated error can be neglected. Altogether this proves that the mentioned controller design method can be applied effectively in hemodialysis machines.

Furthermore, this technique has an important advantage compared to the existing methods [10, 11]. Namely, the safety critical equipment (such as hemodialysis machines) is tested to verify and validate its behavior. By using soft computing methods it often cause a problem to tell the exact behavior in low level in a given case. However, the controller which was designed with the presented method is mathematically well described and the test inputs and test outputs can be created easily. This latter reason and that the LMI design method is generally applicable makes it possible to use in hemodialysis machines not only for fluid control, but for other control purposes as well.

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