Nonlinear Order-Reduced Adaptive Controller for a DC Motor Driven Electric Cart

József K. Tar†, Tamás Haidegger†, Levente Kovács†, Krisztián Kósi‡, Balázs Botka, Imre J. Rudas†
†Antal Bejczy Center for Intelligent Robotics
‡Doctoral School of Applied Informatics and Applied Mathematics
Óbuda University, Budapest, Hungary
Email: {tar.jozsef@nik., rudas@, haidegger@, kovacs.levente@nik.,
kosi.krisztian@phd.}uni-obuda.hu, balazs.botka@gmail.com

***Abstract*—The precise control of a voltage-controlled, DC motor driven electric cart would be a 3rd order task since Classical Mechanics relates the acceleration of the cart to driving torques, while these torques are proportional to the current that cannot be modified abruptly. Abrupt jumps in the control voltage cause abrupt change only in the time-derivative of the motor current. Therefore a 2nd order controller can only approximate the operation of a 3rd order one. Modeling imprecisions further complicate the control task. Furthermore, if the cart has two independently driven main wheels and a supporting caster, no exact trajectory tracking can be prescribed simultaneously for the location of the cart in the (x, y) plane and its rotational orientation (θ) since for three prescribed quantities only two control signals are available. For the tracking error of these components, some kinematically formulated compromise must be applied. The aim of the present paper is to show that the simultaneous problems caused by the order reduction, the modeling errors and the necessary kinematic compromises can be elegantly solved by the adaptive controllers designed on the basis of *Robust Fixed Point Transformations* (RFPT). This statement is substantiated by simulation results.**

I. Introduction

The tasks of solving the motion equations of coupled non-linear systems that are either "free" or are under the effect of some linear or nonlinear controller normally share a common difficulty: in general these solutions cannot be expressed by closed analytical form built up of "elementary functions" being finite in their number.

As a panacea to this problem a particular solution was proposed by Lyapunov in 1892 [1], [2]: by giving up the need of describing the trajectory of the controlled system in details, under special circumstances, definite statements can be done for its *stability*. For this purpose various stability definitions as "common Stability", "asymptotic stability", "global stability", "global asymptotic stability", "exponential stability" can be applied.

As it was exemplified by Lorenz in the 1960s—for a wide class of problems—the appearance of high computational power computers practically cannot provide solution to this problem [3]. In general, the numerical results are "confined" in the bounded interval of the actual computations and cannot be extrapolated outside of this region without further care and considerations.

On this reason, in the fields of "Model-Based Control" and "Adaptive Control" mainly Lyapunov's *direct method* is used to design controllers (e.g., [4], [5], [6], [7], [8], [9], [10], [11]).

Though Lyapunov's method has the great virtue that it normally guarantees *global stability*, it also has certain drawbacks as follows:

- The primary intent of the designer of the controller may be to impose precise restrictions on the tracking error relaxation as the controller "learns" or tunes itself. However, these details are not in the focus of the design and they can be revealed only by numerical computations.
- Normally the Lyapunov function may contain *ample number of arbitrary adaptive control parameters* (mainly among the matrix elements of positive definite symmetric matrices). The global stability can be guaranteed for various settings that have significant effects on the details of the controlled motions. For determining the practically satisfactory setting some optimization can be done even by the use of the means of *evolutionary computations* (e.g., [12], [13]) that normally may mean high computational burden.
- Though it is easy to understand the mathematical essence of Lyapunov's method, its particular applications require very good skills on behalf of the designer.
- The method is built up on *rather satisfactory than necessary conditions*, consequently it normally requires *"too much"*, i.e., it works with more than necessary stipulations.
- These stipulations mainly originate from *formal considerations* and do not allow the method to become "versatile enough". For instance, it was recently shown that slight modification of the parameter tuning rules of the "classic" *Adaptive Inverse Dynamics Controller* and the *Slotine-Li Adaptive Controller*, due to which the tuning rules were not deduced from a Lyapunov function it became possible to combine a modern adaptive technique with the classic parameter learning methods [14], [15].

To evade the above difficulties, an alternative adaptive design method, the *Robust Fixed Point Transformations* (RFPT)-based design was introduced [16]. This method applies a particular iterative learning control in which the iterative sequence is obtained by the use of a contractive map in a

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Banach Space and it converges on the basis of Banach's *Fixed Point Theorem* [17]. Furthermore it places into the focus the realization of a prescribed trajectory tracking error relaxation. In its simplest form it only needs 3 adaptive parameters that can be fixed for many applications. It can guarantee only a bounded basin of convergence that may be left by the system. If it is necessary for maintaining the convergence, one of its parameters can be adaptively tuned by various manners (e.g., [18], [19], [20]). With the introduction of these tuning rules only a few new parameters are introduced that have well identified roles. This design has the advantage that it does not need any precise initial model of the system under control. It can do with a very approximate model: without trying to "amend" this model it adaptively deforms its input via observing the behavior of the controlled system. It can well compensate the simultaneous effects of modeling errors and unknown, directly not observable external disturbances. (Since no model improvement happens, this control permanently needs fresh observations and cannot promise asymptotic stability.)

In the present paper, our aim is to show that the extremely good versatility of the RFPT-based design can be elegantly utilized for solving the adaptive control of caster-supported cart with two, DC motor driven wheels. This problem was intensively researched even in the recent years. For instance in 2002, the conventional technique of dynamic feedback linearization was applied [21]. In 2008, LeBel and Rodrigues applied piecewise-affine parameter-varying control for problem tackling [22]. In 2013, *Switched-Fuzzy* and *Fuzzy-Neural Systems* were applied as problem solutions [23]. Each of these papers intensively applied the Lyapunov technique. In the hereby suggested control, order-reduction and simultaneous compensation of the effects of modeling errors is solved while the kinematically constrained nature of this system is taken into consideration with a simple optimization technique.

II. The Kinematic and Dynamic Models of the Cart

A. The Kinematic Model

The kinematic model was taken from [21]. Assuming that no skidding occurs the (x, y, θ) coordinates of the frame fixed on the road, i.e., the time-derivatives of the position of the center of the cart on the (x, y) plane with Cartesian coordinates and that of its rotational orientation θ are unique functions of the rotational velocities of the wheel axles as

$$\begin{array}{c} \dot{x} = \frac{r}{2}\left(\dot{q}_r + \dot{q}_l\right)\cos(\theta), \\ \dot{y} = \frac{r}{2}\left(\dot{q}_r + \dot{q}_l\right)\sin(\theta), \\ \dot{\theta} = \frac{r}{2D}\left(\dot{q}_r - \dot{q}_l\right), \end{array} \tag{1}$$

in which $r = 0.1\,m$ denotes the radii of the wheels, $D = 1\,m$ means the distances between the wheels and the center of the cart, that is assumed to be on the line connection the centers of the wheels, and q_r and q_l denote the rotation angles of the wheels at the right and the left sides of the cart, respectively. This system is a non-holonomic device in which the pair of variables (q_r, q_l) cannot be used as generalized coordinates of the mechanical system. Via further derivation of (1), the acceleration data can be obtained that are utilized in the dynamic model of the system:

$$\begin{array}{c} \ddot{x} = \frac{r}{2}\left(\ddot{q}_r + \ddot{q}_l\right)\cos(\theta) - \frac{r}{2}\left(\dot{q}_r + \dot{q}_l\right)\sin(\theta)\dot{\theta}, \\ \ddot{y} = \frac{r}{2}\left(\ddot{q}_r + \ddot{q}_l\right)\sin(\theta) + \frac{r}{2}\left(\dot{q}_r + \dot{q}_l\right)\cos(\theta)\dot{\theta}, \\ \ddot{\theta} = \frac{r}{2D}\left(\ddot{q}_r - \ddot{q}_l\right). \end{array} \tag{2}$$

B. Kinematically Formulated Desired Trajectory Tracking for the Given Kinematic Constraints

Assume that the user of the cart should like to kinematically prescribe the trajectory tracking error relaxation by the formula:

$$\begin{array}{c} \left(\Lambda + \frac{d}{dt}\right)^3 \int_{t_0}^{t} \left(X^N(\xi) - X(\xi)\right) d\xi \equiv 0 \\ \Lambda > 0 \end{array}, \tag{3}$$

where $X^N(t)$ is the *nominal trajectory to be tracked*, while $X(t) \stackrel{def}{=} (x, y, \theta)$ is the actual one. This prescription leads to the *desired 2^nd^ time-derivative* that corresponds to a PID-type control as:

$$\begin{array}{c} \ddot{X}^{Des} \stackrel{def}{=} \ddot{X}^N + \Lambda^3 \int_{t_0}^{t} \left[X^N(\xi) - X(\xi)\right] d\xi \\ +3\Lambda^2 \left[X^N(t) - X(t)\right] + 3\Lambda \left[\dot{X}^N(t) - \dot{X}(t)\right] \end{array} \tag{4}$$

resulting exponential error-relaxation for $\Lambda > 0$. Due to the fact that for the *three quantities to be controlled*, i.e., for x, y, and θ we have only two control actions as T_r and T_l, (4) cannot be realized. Some compromise has to be introduced that somehow distributes the tracking error over these quantities. For this purpose a very simple approach was applied in this paper: in each control cycle the *quadratic error*:

$$\begin{array}{c} \Phi(\ddot{q}_r^{Des}, \ddot{q}_l^{Des}) \stackrel{def}{=} \\ = \left(\ddot{x}^{Des} - \ddot{x}\right)^2 + \left(\ddot{y}^{Des} - \ddot{y}\right)^2 + \kappa\left(\ddot{\theta}^{Des} - \ddot{\theta}\right)^2 \\ in\ which \\ \kappa > 0, \\ \ddot{x} = \ddot{x}(q_r, q_l, \dot{q}_r, \dot{q}_l, \ddot{q}_r^{Des}, \ddot{q}_l^{Des}), \\ \ddot{y} = \ddot{y}(q_r, q_l, \dot{q}_r, \dot{q}_l, \ddot{q}_r^{Des}, \ddot{q}_l^{Des}),\ and \\ \ddot{\theta} = \ddot{\theta}(q_r, q_l, \dot{q}_r, \dot{q}_l, \ddot{q}_r^{Des}, \ddot{q}_l^{Des}) \end{array} \tag{5}$$

was minimized according to the variables $(\ddot{q}_r^{Des}, \ddot{q}_l^{Des})$. In this manner, the appearance of complicated Riccati equations that are typical in *optimal control* was simply evaded. By the use of the kinematic equations (5) led to the inversion of a simple $\mathbb{R}^2$ matrix that was solved "by hand". In the use of the dynamic model, the kinematically realizable $(\ddot{q}_r^{Des}, \ddot{q}_l^{Des})$ quantities were taken into consideration.

C. The Dynamic Model of the Cart

The dynamic model can directly be deduced from the laws of Classical Mechanics: the acceleration of the mass center point of the cart with respect to the inertial road-system is proportional to the inertia of the whole system, $M = 20\,kg$, and its acceleration. The angular acceleration of the rotation around the mass center point multiplied by the momentum of the system $I = 10\,kg\cdot m^2$ must yield the rotary torque. It was assumed that M and I cannot be exactly known if the cart carries some work-load. In the simulations, their approximate counterparts $\hat{M} = 25\,kg$, and $\hat{I} = 15\,kg\cdot m^2$ were used for the calculation of the control forces and the exact values we applied for the calculation of the motion of the cart.

Regarding the scaling rules for the motor-wheel axles, if a wheel of radius $r\,[m]$ exerts the torque $T\,[N\cdot m]$ the contact force at the road must be $F = \frac{T}{r}\,[N]$, that with the arm length $D\,[m]$ exerts $FD = \frac{TD}{r}\,[N\cdot m]$ torque that rotates around the mass center point. Therefore it can be written that

$$\begin{aligned} I\ddot{\theta} &= \tfrac{Ir}{2D}\left(\ddot{q}_r - \ddot{q}_l\right) = \tfrac{D}{r}(T_r - T_l),\\ M\begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix} &= \begin{pmatrix}\cos(\theta)\\ \sin(\theta)\end{pmatrix}\tfrac{T_r+T_l}{r}. \end{aligned} \tag{6}$$

Via making the *scalar product* of the 2nd equation in (6) with $(\cos(\theta), \sin(\theta))^T$ and utilizing (2) let us obtain that:

$$\begin{aligned} \tfrac{Mr^2}{2}\left(\ddot{q}_r + \ddot{q}_l\right) &= T_r + T_l\\ \tfrac{Ir^2}{2D^2}\left(\ddot{q}_r - \ddot{q}_l\right) &= T_r - T_l \end{aligned}\ , \tag{7}$$

from which the necessary torques derive for a given pair $(\ddot{q}_r, \ddot{q}_l)$.

D. The Model of The DC Motors

In this case it is assumed that the rotary axles of the DC motors are rigidly connected to that of the wheels by the gear reduction ratio $\nu = 0.1$ as $q_{rl}^{Mot} = \frac{q_{rl}}{\nu}$, $\dot{q}_{rl}^{Mot} = \frac{\dot{q}_{rl}}{\nu}$, and $\ddot{q}_{rl}^{Mot} = \frac{\ddot{q}_{rl}}{\nu}$ (the rotational speed of the motor axles is higher than that of the wheels). Due to the conservation of the energy (the mechanical work) in the coupling cog-wheels also determines the scaling rule for the torques as follows: the $\delta\varphi_1$ rotation of the axle of radius r_1 causes the displacement $\delta s = r_1\delta\varphi_1$ at the connecting cogs. The same displacement has to be done by the cog of the other wheel as $\delta s = r_2\delta\varphi_2$. If the contact force F_1 and its reaction force $F_2 = -F_1$ in absolute value has to make the same mechanical work as transmitted energy, i.e. $|F_1|r_1\delta\varphi_1 = |F_2|r_2\delta\varphi_2$ in which the torques $T_1 \overset{def}{=} |F_1|r_1$ and $T_2 \overset{def}{=} |F_2|r_2$ can be recognized. Therefore, $\frac{\delta\varphi_1}{\delta\varphi_2} = \frac{r_2}{r_1} \overset{def}{=} \nu$, and $\frac{T_1}{T_2} = \frac{\delta\varphi_2}{\delta\varphi_1} = \frac{1}{\nu}$ can be written. By the use of the wheel axles in the motor model taken from [24] it can be written that:

$$\begin{aligned} \nu\ddot{q}_{rl} &= \frac{Q_{rl}^e + \frac{Q_{rl}^{ext}}{\nu} - b\nu\dot{q}_{rl}}{\Theta}\\ \dot{Q}_{rl}^e &= \frac{-RQ_{rl}^e - K^2\nu\dot{q}_{rl} + KU_{rl}}{L} \end{aligned}\ , \tag{8}$$

where *identical motors* were assumed at the LHS and RHS, with the variables and parameters as follows:

- $Q^e\,[N\cdot m]$ is the torque of electromagnetic origin exerted on the motor's axle (it is proportional to the motor current),
- $Q^{ext}\,[N\cdot m]$ is the torque of external origin acting on the wheel's axle, i.e., $Q_{rl}^{ext} = T_{rl}$ in (7),
- $R = 1\,[\Omega]$ is the Ohmic resistance of the motor's coil system,
- $L = 0.5\,[H]$ is its inductivity,
- $\Theta = 0.01\,[kg\cdot m^2]$ denotes the momentum of the rotary part of the motor,
- $b = 0.1\,[N\cdot m\cdot s/rad]$ describes the viscous friction of the motor's axle,
- $K = 0.01$ is the motor's torque coefficient, and
- $U\,[V]$ denotes the motor control voltage.

It is evident from (8) that an *abrupt variation in* U causes an *abrupt variation* only in $\dot{Q}^e$, therefore instead of Q^e only $\dot{Q}^e$ can directly be controlled by U. The "orthodox way" for developing a precise control would require the calculation of the time-derivative of the first equation in (8), in which $\dddot{q}$ could be directly related to U through the 2nd equation. That is the precise control that should be developed for $\dddot{q}$, and therefore for $\dot{T}_{rl}$ instead of T_{rl} that can be calculated directly from the mechanical model in (7).

Instead of following the "orthodox way", we develop an RFPT-based adaptive design for a 2nd order control as it is expounded in the next section.

III. The RFPT-Based Design for Order Reduced Adaptive Controller

If we wish to avoid the development of a 3rd order control, we have to apply some order reduction technique. In the realm of the *Linear Time-Invariant* (LTI) systems, that can be described in the frequency domain by fractional polynomial expressions as *Transfer Functions* the Padé approximation theory [25] can widely be used for order reduction even in the case of fractional order systems of long memory (e.g., [26]). However, in the case of nonlinear systems alternative approaches have to be chosen.

To avoid 3rd order control, the RFPT-based order reduction can be formulated as follows: if $\dot{q}$ is constant, the 2nd equation of the group (8) could describe a *stable linear system* that could exponentially trace the abrupt jumps in U. If the electromagnetic components work considerably faster than the mechanical ones group, (8) could be used for designing abrupt changes in U_{rl} to realize $\ddot{q}_{rl}^{Des}$ in the control cycles. However, this is only an approximation. The role of the RFPT-based adaptive design consists in correcting this preliminary design, together with the effects of the modeling errors and unknown external disturbances. In this approach, $\ddot{q}_{rl}^{Des}$ is computed from (7), but instead of the "exact model" the same equations with the approximate model parameters can be used. By the use of the first equation of the group (8) $Q_{rl}^{e^{Des}}$ is calculated for $\ddot{q}_{rl}^{Des}$. Assuming that $\dot{Q}_{rl}^e \approx 0$ for a given constant, $\dot{q}_{rl}$, the stabilized value of the necessary U_{rl}^{Des} is estimated from the 2nd equation to be:

$$U_{rl}^{Des} \overset{def}{=} \frac{R}{K}Q_{rl}^{e^{Des}} + K\nu\dot{q}_{rl}. \tag{9}$$

The adaptivity is introduced in the above outlined argumentation when $\ddot{q}^{Des} \overset{def}{=} (\ddot{q}_r^{Des}, \ddot{q}_l^{Des})^T \in \mathbb{R}^2$ value is replaced by its adaptively deformed counterpart for control cycle $(n+1)$ as:

$$\begin{aligned} \mathbb{R}^2 \ni e_n &\stackrel{def}{=} \tfrac{\ddot{q}_n - \ddot{q}^{Des}_{n+1}}{\|\ddot{q}_n - \ddot{q}^{Des}_{n+1}\|}, \\ \tilde{B} &\stackrel{def}{=} B_c \sigma\left(A_c \|\ddot{q}_n - \ddot{q}^{Des}_{n+1}\|\right), \\ \ddot{q}^{Req}_{n+1} &\stackrel{def}{=} (1+\tilde{B})\ddot{q}^{Req}_n + \tilde{B} K_c e_n, \end{aligned} \tag{10}$$

in which A_c, B_c and K_c are adaptive control parameters, $\mathbb{R}^2 \ni \ddot{q}_n$ is the *observed response* at cycle n, and:

$$\sigma(x) \stackrel{def}{=} \frac{x}{1+|x|}. \tag{11}$$

Evidently, if $\ddot{q}_n = \ddot{q}^{Des}_{n+1}$, i.e., when we found the appropriate deformation, $\ddot{q}^{Req}_{n+1} = \ddot{q}^{Req}_n$, that is the solution of the control task, the fixed point of the mapping defined in (10). For convergence, this mapping must be made *contractive*. For this purpose, normally $B_c = \pm 1$, a very big $|K_c|$, and an appropriately small $A_c > 0$ value have to be chosen. For the details see e.g. [19] and [20]. In this paper, this issue will not be researched in details.

IV. Simulation Results

The simulations were performed by using SCILAB 5.4.1 for LINUX and its graphical tool, called XCOS. These software are open-access via the [27]. These were primarily developed for the needs of higher education in France [28], yet are also very for solving optimization problems. They provide interfaces to other, free and efficient software [29]. It offers various numerical integrators for *Ordinary Differential Equations*. In the simulations we used the *Livermore Solver for solving Ordinary Differential Equations*, an option abbreviated as *LSodar* that applies an automatic switching for stiff and non-stiff problems. It also uses variable step size and combines the *(Backward Differentiation Formula (BDF)* and *Adams* integration methods. The stiffness detection is done by step size attempts in both cases. In (10), the element called *continuous time delay* was used to utilize the "past values" in the iteration. Normally the necessary time-delay depends on the dynamics of the motion to be tracked and it also directly influences the available tracking precision. The discrete time-resolution (i.e., the cycle-time of the controller) was $\delta t = 10^{-3}\,s$.

To achieve useful results, the allowable step-size was limited to 10^{-2} in the simulations by setting the solver. One of the advantages of the RFPT-based methods is that they can work with relatively small Λ values. In our case $\Lambda = 1\,s^{-1}$ and $\Lambda = 0.5\,s^{-1}$ values were applied in (4). The adaptive control parameters were set as $B_c = -1$, $K_c = 10^8$ and $A_c = 5 \times 10^{-9}$, and no tuning for A_c was necessary. To check the abilities of the controller a "slalom-type" nominal trajectory was chosen with an appropriate orientation θ^N, that corresponded to that of the actual tangent of the trajectory.

In Figs. 1 and 2 the results for the trajectory tracking can be seen for $\Lambda = 0.5\,s^{-1}$ and $\Lambda = 1\,s^{-1}$ values were employed in (4). The *nominal trajectory* intentionally contained relatively sharp turns that are more appropriate to test the control method than the relatively smooth ones. As it was expected, greater Λ caused "tighter", i.e., more precise tracking. The slow relaxation of the orientation error is quite illustrative. According to Fig. 3—describing the rotary speeds of the wheels—it is evident that significant differences between the values belonging to the greater and lesser Λ parameters are only in the initial transient phase. The appropriate control voltages are described in Fig. 4. It is well shown that following a hectic transient, initial section the voltages vary quite "smoothly", depending on the needs of the nominal trajectory and the system's dynamics. As it could be expected, for greater Λ, greater initial fluctuations pertain.

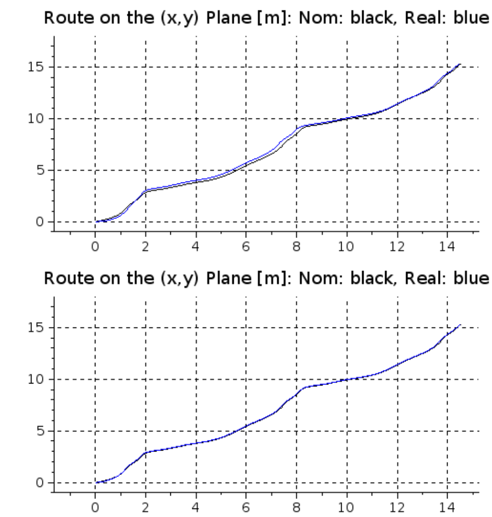

Fig. 1. Tracking of the trajectory in the (x, y) plane for $\Lambda = 0.5\,s^{-1}$ (upper chart) and $\Lambda = 1\,s^{-1}$ (lower chart). [The nominal trajectory: black line, the simulated one: blue line.]

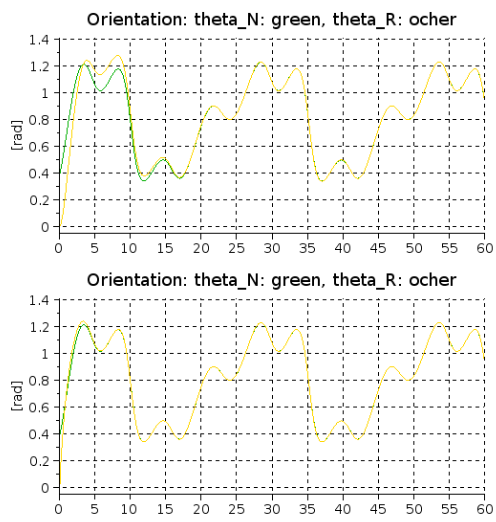

Fig. 2. Tracking of the trajectory for the orientation θ for $\Lambda = 0.5\,s^{-1}$ (upper chart) and $\Lambda = 1\,s^{-1}$ (lower chart) [The nominal trajectory: green line, the simulated one: ocher line, time in $[s]$ units in the horizontal axes]

To reveal the operation of the adaptive controller Figs. 5, 6 and 7 describe the kinematically calculated "Desired", the adaptively deformed "Required", and the simulated "Realized"

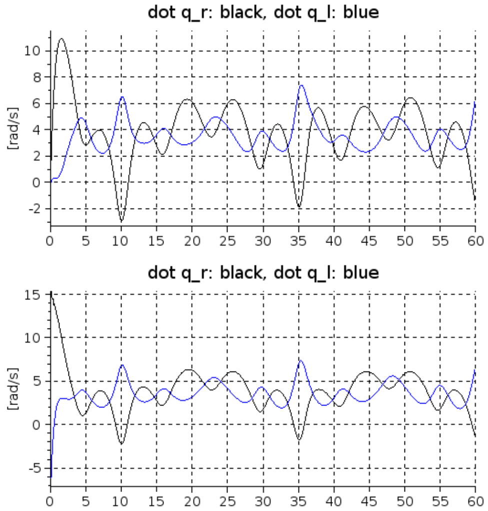

Fig. 3. The rotary speed of the wheels for $\Lambda = 0.5\,s^{-1}$ (upper chart) and $\Lambda = 1\,s^{-1}$ (lower chart). [$\dot{q}_r$: black line, $\dot{q}_l$: blue line, time in $[s]$ in the horizontal axes.]

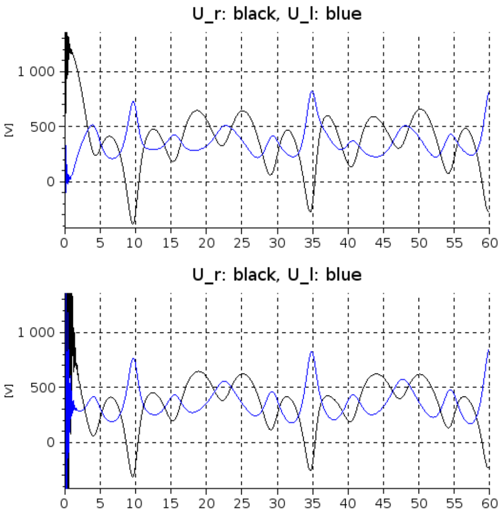

Fig. 4. The control voltages versus time in $[s]$ for $\Lambda = 0.5\,s^{-1}$ (upper chart) and $\Lambda = 1\,s^{-1}$ (lower chart). [U_r: black line, U_l: blue line.]

values for $\ddot{q}_{rl}$. The extent of the adaptive deformation is quite significant, i.e., the "Desired" and the "Required" values are quite different, but the "Desired" values are precisely approximated by the "Realized" ones.

V. Conclusions

In this paper the possible application of the *Robust Fixed Point Transformations* (RFPT)-based controller design method was demonstrated through the control of two, independent DC motor driven electric carts ("a wheeled mobile robot"). The kinematically purely designed tracking policy was shown to be able to well be approximated by a 2nd order nonlinear adaptive controller in spite of the fact that a precise control policy could require 3rd order control. This suggests kinematically realizable compromise for the three components of the

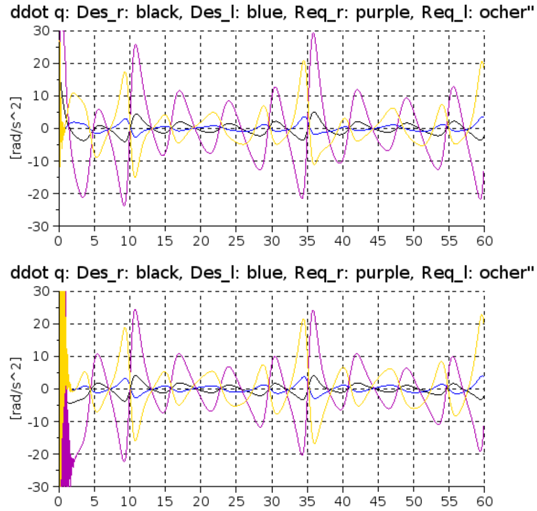

Fig. 5. The *"Desired"* and the *adaptively deformed "Required"* second time-derivatives of the wheels' axles versus time in $[s]$ units for $\Lambda = 0.5\,s^{-1}$ (upper chart) and $\Lambda = 1\,s^{-1}$ (lower chart). [$\ddot{q}_r^{Des}$: black line, $\ddot{q}_r^{Req}$: purple line, $\ddot{q}_l^{Des}$: blue line, $\ddot{q}_l^{Req}$: ocher line.]

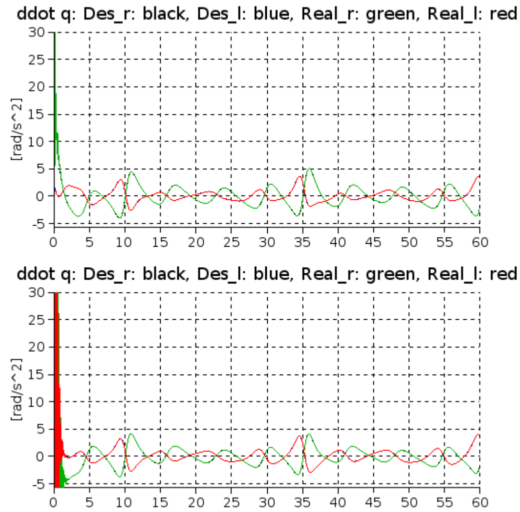

Fig. 6. The *"Desired"* and the *simulated "Real"* second time-derivatives of the wheels' axles versus time in $[s]$ units for $\Lambda = 0.5\,s^{-1}$ (upper chart) and $\Lambda = 1\,s^{-1}$ (lower chart). [$\ddot{q}_r^{Des}$: black line, $\ddot{q}_r^{Real}$: green line, $\ddot{q}_l^{Des}$: blue line, $\ddot{q}_l^{Real}$: red line.]

tracking error by requiring desired 2nd time-derivatives for the wheel axes $\ddot{q}_{rl}^{Des}$.

The adaptive abilities of the RFPT design allows simultaneous compensation of the consequences of the modeling errors and the order reduction in the control. This order reduction technique is quite different to the traditional solutions based on the frequency domain, that is typically applicable only for LTI systems. In general, the RFPT-based design has the great advantage over the Lyapunov function based techniques that it is very simple. While the application of Lyapunov's 2nd method, where the negative time-derivative of the Lyapunov function is proved took a lot of effort. Our method simply

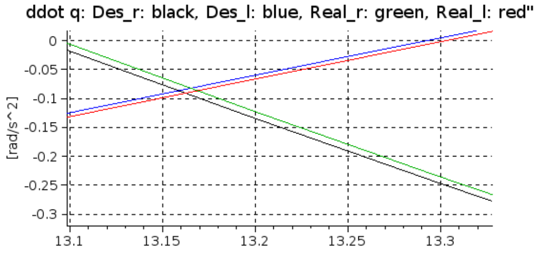

Fig. 7. The *"Desired"* and the *simulated "Real"* second time-derivatives of the wheels' axles versus time in $[s]$ for $\Lambda = 1\,s^{-1}$ [$\ddot{q}_r^{Des}$: black line, $\ddot{q}_r^{Real}$: green line, $\ddot{q}_l^{Des}$: blue line, $\ddot{q}_l^{Real}$: red line, zoomed excerpts.]

applies iterative control sequences. These are obtained from a contractive map over a linear, normed, complete metric space (Banach space), for which from Banach's fixed point theorem immediately follows: the control sequence has to converge to the fixed point of this map. Our map is constructed that its fixed point is the solution of the control task. For this purpose, neither precise nor complete system model is needed.

Our further aim is to investigate the applicability of this adaptive order reduction technique for higher order problems that typically occur whenever the driving forces can act on the controlled system through a chain of significantly deformable components.

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